University of Natural Resources and Life Sciences, Vienna
Department of Water, Atmosphere and Environment

Transient pipe flow (hydraulic surge)

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Outline I

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Governing equations

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Newtons' second law of motion:

$$F = m \cdot a = m \cdot \frac{\partial v}{\partial t} \Rightarrow \frac{F}{m} = \frac{\partial v}{\partial t} \tag{1}$$

$$F = \Delta p \cdot A \tag{2}$$

$$\Rightarrow \frac{\partial v}{\partial t} = \frac{\Delta p \cdot A}{m} = \frac{\Delta p}{\rho \cdot \mathrm{d}x} \Rightarrow \frac{\partial v}{\partial t} = \frac{1}{\rho} \cdot \frac{\partial p}{\partial x} \tag{3}$$

Governing equations

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Equation for the solution of transient pipe flow (Hanif, 1986):
Momentum conservation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial h}{\partial x} + \frac{\lambda}{2d} \cdot v|v| = 0 \tag{4}$$

Mass conservation:

$$\frac{\partial h}{\partial t} + \frac{a^2}{g}\frac{\partial v}{\partial x} + v \cdot \left(\frac{\partial h}{\partial x} + \sin\Theta\right) = 0 \tag{5}$$

Governing equations

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Discretization of equations (4) and (5) neglecting non-linear terms (Hanif, 1986):

$$\begin{aligned} v_i^{n+1} =& \frac{1}{2} \cdot \left(v_{i-1}^n + v_{i+1}^n + \frac{g}{a} \cdot (h_{i-1}^n - h_{i+1}^n) \right. \\ & \left. - \frac{\lambda \cdot \Delta t}{2d} \cdot (v_{i-1}^n \cdot |v_{i-1}^n| + v_{i+1}^n \cdot |v_{i+1}^n|) \right) \end{aligned} \tag{6}$$

$$\begin{aligned} h_i^{n+1} =& \frac{1}{2} \cdot \left(h_{i-1}^n + h_{i+1}^n + \frac{a}{g} \cdot (v_{i-1}^n - v_{i+1}^n) \right. \\ & \left. - \frac{a}{g} \cdot \frac{\lambda \cdot \Delta t}{2d} \cdot (v_{i-1}^n \cdot |v_{i-1}^n| + v_{i+1}^n \cdot |v_{i+1}^n|) \right) \end{aligned} \tag{7}$$

Boundary conditions

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Characteristic equation according to Hanif (1986):

$$
\begin{aligned}
C_1 &= v_1^n - \frac{g}{a} \cdot h_1^n - \frac{\lambda \cdot \Delta t}{2d} \cdot v_1^n \cdot |v_1^n| && (8)\\
C_2 &= \frac{g}{a} && (9)\\
C_3 &= v_{m-1}^n - \frac{g}{a} \cdot h_{m-1}^n - \frac{\lambda \cdot \Delta t}{2d} \cdot v_{m-1}^n \cdot |v_{m-1}^n| && (10)\\
C_v &= \frac{v_0^0 \cdot v_0^0}{C_2 \cdot h_L^0} && (11)\\
C_4 &= \tau^2 \cdot C_v && (12)
\end{aligned}
$$

using:

$$
\tau = \begin{cases} 1 - \frac{t}{T_S} & t < T_S \\ 0 & \text{sonst} \end{cases} \qquad (13)
$$

T_S .. closing time $[T_S] = \mathrm{s}$

Boundary conditions

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Reservoir with water level H at inlet

$$v_0 = C_1 + C_2 \cdot H \tag{14}$$

$$h_0 = H \tag{15}$$

Boundary at end of pipe:

$$v_L = \begin{cases} 0{,}5 \cdot C_4 \cdot \left(-1 + \sqrt{1 + 4 \cdot \frac{C_3}{C_4}}\right) & t < T_S \\ 0 & \text{sonst} \end{cases} \tag{16}$$

$$h_L = \frac{-(v_L - C_3)}{C_2} \tag{17}$$

Exercise

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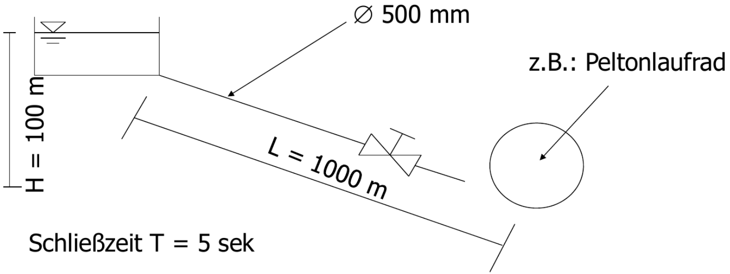

Fig.: Exercise on the hydraulic surge

Exercise: Numerical method and implementation

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1. Does the provided code use an explicit or an implicit scheme?
   → explicit
2. How are time steps and spatial discretization defined?
   → Loop over time with increment `dt`; index in space `j`
3. Where do initial and boundary conditions come in?
   → initial conditions lines 119 to 128;
   → boundary conditions lines 142 to 148
4. Which initial conditions are used?
   → constant velocity along the entire pipe;
   → continuous pressure loss through hydraulic losses from quadratic from the Darcy-Weisbach equation;
5. Which boundary conditions are implemented?
   → see section 1

Exercise: Numerical method and implementation

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Inputfile `Input.txt` with standard values:

```
d         0.5
s         0.005
Ew        2.E9
Er        2.E11
rho       1000.0
g         9.81
L         1000.0
H         100.0
lambda    0.02
Ts        5.0
T         100.0
nout      1
output    output.txt
```

start program on Windows command line: `druckstoss.exe`

Exercise: Hydraulic surge I/II

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1. What is the maximum and minimum pressure head using the standard input values?
   - $\rightarrow$ $h_{\mathrm{max}} = 283\,\mathrm{m},\ h_{\mathrm{min}} = -65\,\mathrm{m}$
   - $\rightarrow$ $v_0 = 6{,}92\,\mathrm{m\,s^{-1}},\ a = 1\,000{,}0\,\mathrm{m\,s^{-1}}$
2. How do the extreme values compare to an estimation based on the rigid water column theory?
   - $\rightarrow$ $h_{\mathrm{max}} = 272\,\mathrm{m},\ h_{\mathrm{min}} = -73\,\mathrm{m}$

$$\frac{\Delta H_{\mathrm{max}}}{H} = \frac{K}{2} \pm \sqrt{K + \frac{K^2}{4}} \qquad \text{using}: K = \left(\frac{L \cdot v_0}{g \cdot H \cdot T_s}\right)^2 \tag{18}$$

Exercise: Hydraulic surge II/II

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3. At which point along the pipe do the above calculated values occur?
   - $\rightarrow$ first segment after the reservoir, 5 m after the reservoir assuming 20 segments total;
4. What are the values for maximum and minimum pressure head at the valve? What can be concluded?
   - $\rightarrow$ $h_{\max} = 717$ m, $h_{\min} = -457$ m
5. Which value does pressure at the valve approach with time? Why?
   - $\rightarrow$ hydrostatic pressure of 100 m (extend simulation time `T`)
6. How does the pressure surge develop without damping through wall friction?
   - $\rightarrow$ pressure head oscillates with constant amplitude between 563 m and -363 m

Exercise: Hydraulic surge

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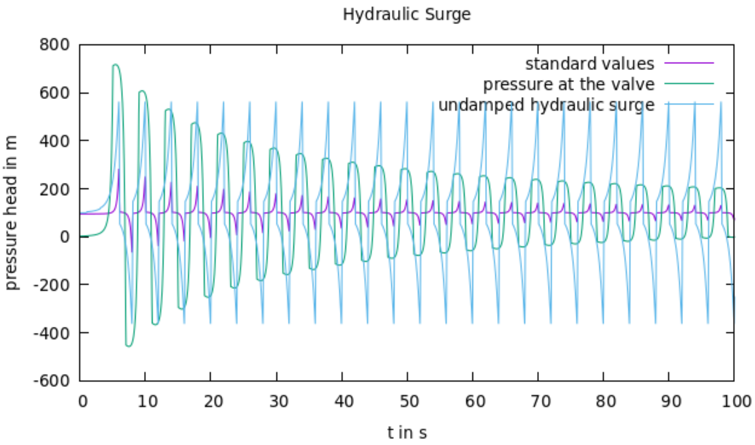

Exercise: Modifications of the system I/IV

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1. How does the maximum pressure head change when the pipe diameter is doubled (0,5 m to 1 m)?
   - → $h_{\max} = 289\,\mathrm{m}$, $h_{\min} = -73\,\mathrm{m}$
   - → pressure comparable to small pipe diameter as pipe diameter also affects velocity and flow rate:
     - $v_0 = 9{,}67\,\mathrm{m\,s^{-1}}$, $a = 816{,}5\,\mathrm{m\,s^{-1}}$
   - → slower damping of the pressure wave due larger mass in motion
2. How does a decrease of the pipe length by one halve affect the maximum pressure (1 000 m to 500 m)?
   - → $h_{\max} = 214\,\mathrm{m}$, $h_{\min} = -7\,\mathrm{m}$
   - → notable reduction of the maximum pressure;

Exercise: Modifications of the system II/IV

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3. How does the increase of the closing time of the valve affect the maximum pressure (5 s to 10 s)?
   - → $h_{\mathrm{max}} = 200\,\mathrm{m},\ h_{\mathrm{min}} = 9\,\mathrm{m}$
   - → notable reduction of the maximum pressure
4. What happens when the closing time is further increased to 60 s?
   - → $h_{\mathrm{max}} = 106\,\mathrm{m},\ h_{\mathrm{min}} = 95\,\mathrm{m}$
   - → strong reduction of the maximum pressure, best option to decrease hydraulic surge
5. How does the hydraulic surge develop when the valve is suddenly closed?
   - → $h_{\mathrm{max}} = 754\,\mathrm{m},\ h_{\mathrm{min}} = -479\,\mathrm{m}$
   - → very high maximum values for a short period of time, high dynamic stress on material;

Exercise: Modifications of the system III/IV

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6. Can hydraulic surge be reduced when initial (hydrostatic) pressure is increased?
   - pressure surge with 200 m in the reservoir:

   → $h_{\max} = 428\,\mathrm{m}$, $h_{\min} = 1\,\mathrm{m}$

   → pressure head slightly lower in relation to initial pressure (factor 2,14 instead of 2,83)

   → negative pressure values can be avoided through higher initial (hydrostatic) pressure

Exercise: Modifications of the system IV/IV

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7. What would be the values of maximum and minimum pressure at the valve, when initial velocity is reduced to $1\,\mathrm{m\,s^{-1}}$? How do these values compare to simplified theory?
   - $\rightarrow$ $h_{\max} = 129\,\mathrm{m}$, $h_{\min} = 80\,\mathrm{m}$
   - $\rightarrow$ relatively slow damping of the pressure wave because of low losses due to friction
   - $\rightarrow$ $\Delta H = 20\,\mathrm{m}$ according to theory; matches the numerical solution quite well with the exception of the first peak which occurs through superposition during flow time;

Exercise: Modifications of the system

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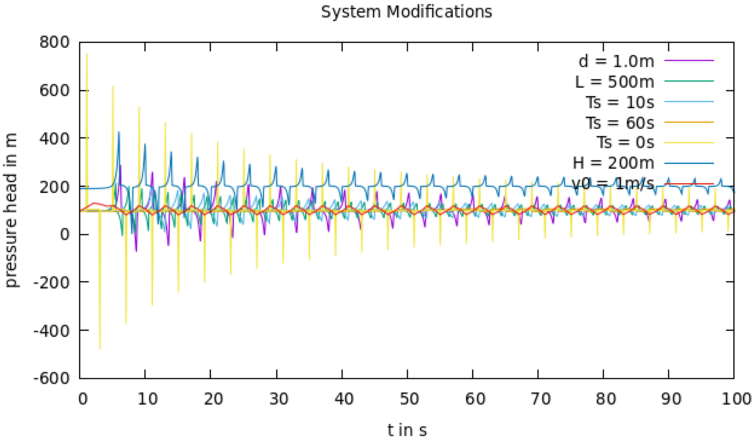

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Literatur I

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Hanif, C. M. (1986). 'Boundary conditions for analysis of waterhammer in pipe systems'. Master Thesis. The University of British Columbia.